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Axiomatic derivation of a between-group stratification index for ordinal health and well-being data

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Abstract

The paper provides an axiomatic characterisation of the Allanson (2017a) headcount stratification index, which provides a summary measure of the extent of differences between population groups that is directly applicable to ordinal or categorical outcome data on individual health status or well-being.

Keywords: headcount stratification; health; well-being; ordinal data

JEL codes: D63; I14; I31

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1. Introduction

Many key questions in health policy require an evaluation of differences in the distribution of individual welfare across population groups, with systematic disparities in health and well-being (simply 'health' hereafter) outcomes generally deemed to be socially inequitable to the extent that they are avoidable by reasonable action to tackle the circumstances that limit the chances of people in disadvantaged groups to live longer, healthier lives. For example, regional disparities in population health outcomes, most notably life expectancy and mortality rates, have been an object of official concern in the United Kingdom since at least the publication of the Black (1980) report. However, investigation of between-group variation in other health outcomes, such as self-assessed health and subjective well-being, has been held back by a lack of appropriate methods to deal with the qualitative nature of such measures, paralleling similar problems in "the evaluation of the inequality in the distribution of health status across individuals in a population" (Allison and Foster, 2004, p.505). In particular, Kobus and Miłos (2012) show that only some inequality indices for ordinal data are groupwise decomposable and only then under the highly restrictive condition that all subgroups share a common median. The main contribution of this paper is to provide an axiomatic characterisation of the Allanson (2017a) headcount stratification index which offers an alternative approach to capturing the extent of ordinal health differences between groups in terms of the average absolute difference in the chances of being in better rather than worse health as a result of being a member of one group rather than another. This measure is equal to twice the between-group absolute Gini index for binary health status indicators but is also well-defined for polytomous categorical variables.

The concept of stratification is deeply embedded within sociology, most notably in relation to the analysis of social class, but has only become established in the economics literature following the seminal article by Yitzhaki and Lerman (1991). Key to our approach is the idea that stratification, unlike segregation, implies a hierarchical ordering of population groups, which for the univariate or 'pure' measurement of health stratification will be by some measure of population health status. If health was cardinally measurable then, following Allanson (2018), this ordering might be in terms of equally distributed equivalent health, which reduces to ranking groups in order of mean health in the absence of inequality aversion. However, if the health measure is ordinal then this criterion is inoperable and some other basis must be found for the comparative evaluation of health profiles across groups. We assume that group health profiles are compared on a pairwise basis, independently of the health profiles of other groups, and make use of a theorem in Dubois et al. (2003) that implies under certain mild restrictions that only a probability-based dominance rule can serve this purpose if health profile preferences are characterised by ordinal invariance. We proceed to define a measure of the pairwise disparity between group health profiles consistent with this ordering criterion and then derive the stratification index as a population-weighted mean of pairwise disparities.

2. Comparative evaluation of the population health profiles of groups.

Consider some population that consists of $G \geq 2$ mutually exclusive and exhaustive groups. The population size and share of group g $(g = 1,....G)$ are given as n_g and $p_g = n_g / N$ respectively, where $N = \sum_g n_g$ is the total population size. Let Ω be the weakly ordered, finite set of potential health outcomes h at the individual level. For example, the set of self-assessed health states in survey questionnaires is commonly defined as Ω = {Very good, Good, Fair, Bad, Very bad}. The health profile of group g is a vector of the form $H_g = (h_{1g}, \cdots, h_{n_g g})$, where h_{ig} is the health status of the i'th person, and the health profile of the whole population $H_U = (H_1, H_2, \ldots H_G)$ is obtained as the union over all groups. We denote the probability that the health of a randomly chosen individual from group g' is at least as good as that of an individual chosen at random from group g as $\mathrm{P}(H_{g'} \geq H_g)$.

The axiomatic derivation of the probability-based dominance rule rests on a set of assumptions concerning the nature of preferences over the health profiles of groups. For any pair of groups g and g', let the binary preference relation $H_{g'} \succsim H_g$ denote that "the health profile of group g' is at least as good as that of group g", where $\succsim$ is composed of a strict preference relation $\succ$ and an indifference relation $\sim$ as is usual. Following Dubois et al. (2003), this relation is assumed to be complete over homogeneous groups, such that if the health of everyone in groups g and g' is h and h' respectively then $H_{g'} \succsim H_g$ if $h' \geq h$, and reflexive. Moroever, Ω is assumed to contain at least two health states h and h' for which $H_{g'}$ is strictly preferred to H_g for some pair of homogeneous groups. Finally it is assumed that the relation is ordinally invariant such that, for any pair of groups g and g', changes in individual health outcomes that preserve the ranking of all individuals in the combined health distribution of the two groups will not affect whether the health profile of g' is judged to be better, the same or worse than that of g. Formally, let $(H_g, H_{g'}) \equiv (H_g^*, H_{g'}^*)$ denote that the health profile pairs $(H_g, H_{g'})$ and $(H_g^*, H_{g'}^*)$ are ordinally equivalent in the sense that one pair can be obtained from the other by an order-preserving transformation of individual health outcomes in the combined profile, then ordinal invariance implies that for all $g, g' \in G$, $\left((H_g, H_{g'}) \equiv (H_g^*, H_{g'}^*)\right) \Rightarrow \left((H_{g'} \succsim H_g) \Leftrightarrow (H_{g'}^* \succsim H_g^*)\right)$. Ordinal invariance is the key property for the comparison of group health profiles if data on health outcomes is qualitative in nature since ranking individuals in the health distribution only requires knowledge of the ordering of health outcomes.

Given this axiomatic framework, Theorem 1 in Dubois et al. (2003, p.232), together with the associated Corollary 1, imply that the only basis for the comparative evaluation of pairs of health profiles is a probability-based dominance rule, i.e. $H_{g'} \succsim H_g$ if and only if $\mathrm{P}(H_{g'} \geq H_g) \geq \mathrm{P}(H_g \geq H_{g'})$. Intuitively, the comparative evaluation of the two profiles may be seen to involve the comparison of the health of each member of $H_{g'}$ with that of every member of H_g. The probability-based dominance rule consists of (strictly) preferring $H_{g'}$ to H_g whenever the number of pairwise comparisons in which the $H_{g'}$ member's health is at least good as that of the H_g member is greater than the number of comparisons in which the opposite condition holds. This pairwise majority rule is rational within a purely qualitative framework in which nothing can be said about the size of any health difference that may exist between any pair of individuals. See Dubois et al. (2003) for a formal proof. [2]

The probability-based dominance rule is formally equivalent to the concept of statistical preference introduced by De Schuymer et al. (2003). Specifically, the health profile of one group may be said to be weakly statistically preferred to that of another if the (strictly) healthier of any randomly matched pair of individuals from the two groups is as least as likely to be from the first than the second group. Weak statistical preference provides a generalisation of weak first-degree stochastic dominance, since the latter implies the former, but not vice versa (De Baets and De Mayer, 2007). Statistical preference will order every pairs of groups, whereas stochastic dominance may not, so it will always be possible to say whether one group health profile is better, worse or equivalent to another. However, the

[2] Groups (g, g') in this paper are equivalent to acts (f, g) in Dubois et al. (2003), potential health outcomes h to consequences X, and probability measures define the likelihood of events with the probability that individual i from group g is randomly matched with i' from g' equal to $1/n_g n_{g'}$ for all $i \in g$ and $i' \in g'$.

resultant ordering of groups need not necessarily be transitive: for example, if $H_g = (5,2,2)$, $H_{g'} = (3,3,3)$ and $H_{g''} = (4,4,1)$, where higher scores imply better health, then $\mathrm{P}(H_{g'} > H_g) = 2/3$, $\mathrm{P}(H_{g''} > H_{g'}) = 2/3$ and $\mathrm{P}(H_g > H_{g''}) = 5/9$ thereby implying $H_{g'} \succ H_g$, $H_{g''} \succ H_{g'}$ but also $H_g \succ H_{g''}$.

3. Axiomatic characterisation of the pairwise disparity between group health profiles

The 'natural' measure of the pairwise disparity between group health profiles associated with the statistical preference criterion is:

$$\left|\Delta_{gg'}\right| = \left|\mathrm{P}\left(H_{g'} \geq H_g\right) - \mathrm{P}\left(H_g \geq H_{g'}\right)\right| = \left|\mathrm{P}\left(H_{g'} > H_g\right) - \mathrm{P}\left(H_g > H_{g'}\right)\right| \tag{1}$$

where the directional measure $\Delta_{gg'}$ is formally equivalent to both the 'distance ratio' *D0* in Dagum (1980) and the first-order 'discrimination index' Δ_1 in Le Breton *et al.* (2008) if group g' has the better profile of the two groups. The absolute difference measure $\left|\Delta_{gg'}\right|$ takes a minimum value of zero when $\mathrm{P}\left(H_{g'} \geq H_g\right) = \mathrm{P}\left(H_g \geq H_{g'}\right)$, although this does not necessarily imply that the two health profiles are identical, and a maximum value of one when the least healthy member of one group is strictly healthier than the healthiest member of the other one. $\left|\Delta_{gg'}\right|$ is also symmetric since $\Delta_{gg'} = -\Delta_{g'g}$ by construction. Nevertheless, $\left|\Delta_{gg'}\right|$ is not a metric as it may violate the triangle inequality: for example, if $H_g = (2,2)$, $H_{g'} = (3,3)$ and $H_{g''} = (4,1)$ then $\left|\Delta_{gg'}\right| = 1 > \left|\Delta_{gg''}\right| + \left|\Delta_{g''g'}\right| = 0 + 0.$

Krantz et al. (1971, pp.170-173) present a set of conditions for the representation of a binary relation such as $\mathrm{P}(H_{g'} \geq H_g) \geq \mathrm{P}(H_g \geq H_{g'})$ in terms of absolute differences, where it is readily apparent that the probability-based dominance rule satisfies these axioms. In particular, for any $\alpha \in (0,1)$, the profile $H_b = \alpha H_g + (1-\alpha) H_{g'}$ lies between H_g and $H_{g'}$ in the sense that the disparity between H_g and $H_{g'}$ is at least as great as that between H_b and either H_g or $H_{g'}$: indeed $\left|\Delta_{gg'}\right| = \left|\Delta_{gb}\right| + \left|\Delta_{bg'}\right|$ by definition. Krantz et al. prove that absolute-difference measurement structures are unique up to a scale factor so the imposition of a maximum value of 1 as an additional normalisation condition serves to fully characterise $\left|\Delta_{gg'}\right|$.

4. Axiomatic characterisation of the headcount stratification index

Let $S(H_U)$ denote a cardinal headcount stratification index that is well-defined for any population health profile H_U and provides a measure of group separation that satisfies a between-group exchange principle, where this fundamental property is seen to play the same central role in the measurement of stratification as the Pigou-Dalton principle of transfers in the measurement of inequality (see Bleichrodt and van Doorslaer, 2006). The univariate version of this principle holds that an exchange of group membership between an individual from one group and an individual in worse or equal health from some other group with a worse health profile will not lead to an increase in stratification provided that the exchange does not affect the ordering of groups. Let $H_U^X = (H_1, \ldots, H_{g'}^X, \ldots, H_{g''}^X, \ldots, H_G)$ denote a health profile with an identical ordering of groups by population health to H_U, where H_U^X is obtained from H_U by an exchange in group membership between an individual with health h^+ from group g'' and someone with health h^- from group g'. The exchange principle then implies that $S(H_U) \geq S(H_U^X)$ for all $g \in G$, $H_{g''} \succ H_{g'}$, $h^+ \geq h^-$. In particular, the principle follows naturally from the positive association of individual and population health values (cf. Arrow, 1951) if the population consists only of the two groups g'' and g', since the disparity between the health profiles of the two groups may be expected to fall, and cannot rise, as a result of the exchange given that $H_{g''} \succ H^X_{g''}$ and $H^X_{g'} \succ H_{g'}$ by definition if $h^+ \geq h^-$.

We proceed to define $S(H_U)$ as a population-weighted mean of the set of absolute difference measures $\left|\Delta_{gg'}\right|$. It is natural to assume that $S(H_U)$ is symmetric with respect to both individuals and groups conditional on group membership, such that it is invariant to the permutation both of any two individuals within any group and of any two groups within the population. Consistency in aggregation then implies (see Diewert, 1993) that $S(H_U)$ must be written as a separable mean of the form:

$$\begin{aligned} S(H_U) &= \phi^{-1}\left(\sum_{g=1}^{G}\sum_{g'=1}^{G} p_g p_{g'}\,\phi\left(\left|\Delta_{gg'}\right|\right)\right) \\ &= \phi^{-1}\left(\sum_{g=1}^{G} p_g \phi\phi^{-1}\left(\sum_{g'=1}^{G} p_{g'}\phi\left(\left|\Delta_{gg'}\right|\right)\right)\right) = \phi^{-1}\left(\sum_{g=1}^{G} p_g \phi\left(S_g\left(H_U\right)\right)\right) \end{aligned} \quad (2)$$

where ϕ is a continuous, increasing function of one variable and $S_g\left(H_U\right)$ provides a symmetric measure of the contribution of group g to overall stratification. $S(H_U)$ is a unit free measure that is invariant to rank-preserving transformations of health outcomes but sensitive to any change in individual health status within the population unless the change is over some health range occupied exclusively by members of the same group as the individual.

Further structure is imposed on ϕ by the between-group exchange principle. In particular, if the ordering of groups is transitive then ϕ must be linear if it is a differentiable function. To see the call for this condition note that the membership exchange envisaged by the principle will not only lead to a change in pairwise stratification between the two groups involved in the swap, g'' and g', but also between this pair of groups and every other group. Let $\partial\left|\Delta_{ab}\right|$ denote the change in pairwise stratification between any two groups a and b as a result of the membership exchange then the principle requires that:

$$\partial\phi(S)=\sum_{a=1}^{G}p_a\partial\phi(S_a)=\sum_{a=1}^{G}p_a\left(\sum_{b=1}^{G}p_b\partial\phi(|\Delta_{ab}|)\right)$$
$$=\sum_{a=1}^{G}p_a\left(\sum_{b=1}^{G}p_b\left\{\phi(|\Delta_{ab}|+\partial|\Delta_{ab}|)-\phi(|\Delta_{ab}|)\right\}\right)\leq 0 \tag{3}$$

where:

$$\partial\left|\Delta_{g'g''}\right|=\partial\left|\Delta_{g''g'}\right|=\operatorname{sgn}(g'-g'')\left(\frac{\theta_{g'}}{np_{g''}}\right)+\operatorname{sgn}(g'-g''))\left(\frac{\theta_{g''}}{np_{g'}}\right)=-\left[\left(\frac{\theta_{g'}}{n_{g''}}\right)+\left(\frac{\theta_{g''}}{n_{g'}}\right)\right];$$

$$\partial\left|\Delta_{ag''}\right|=\operatorname{sgn}(a-g'')\left(\frac{\theta_a}{np_{g''}}\right);a\neq g',g''; \qquad \partial\left|\Delta_{ag'}\right|=\operatorname{sgn}(g'-a)\left(\frac{\theta_a}{np_{g'}}\right);a\neq g',g''$$

$$\partial\left|\Delta_{ab}\right|=\partial\left|\Delta_{ba}\right|=0;\ a,b\neq g',g''; \qquad \partial\left|\Delta_{aa}\right|=0;\forall a\in G;$$

$$\theta_a=\left(P\left(H_a=h^-\right)+2\sum_{h=h^-+1}^{h^+-1}P\left(H_a=h\right)+P\left(H_a=h^+\right)\right)\geq 0;\forall a\in G;$$

and $\partial\phi(S)$ and $\partial\phi(S_g)$ are the changes in the transformed overall and groupwise indices respectively.

For (3) to hold for any possible partition of the population not only requires that $\partial\left|\Delta_{g'g''}\right|$ is non-positive, which will be true by definition, but also that for all other groups $g\neq g',g''$:

$$\partial\phi(S_g)=p_{g''}\left\{\phi\left(\left|\Delta_{gg''}\right|+\partial\left|\Delta_{gg''}\right|\right)-\phi\left(\left|\Delta_{gg''}\right|\right)\right\}+p_{g'}\left\{\phi\left(\left|\Delta_{gg'}\right|+\partial\left|\Delta_{gg'}\right|\right)-\phi\left(\left|\Delta_{gg'}\right|\right)\right\}\leq 0;g\neq g',g'' \tag{4}$$

which may in turn be approximated to an infinite order, assuming differentiability, as the sum of two signed Taylor-series expansions:

$$\partial\phi(S_g)$$
$$\approx p_{g''}\operatorname{sgn}(g-g'')\left\{\sum_{j=1}^{\infty}\frac{\phi^j\left(\left|\Delta_{gg''}\right|\right)}{j!}\left(\frac{\theta_g}{np_{g''}}\right)^j\right\}+p_{g'}\operatorname{sgn}(g'-g)\sum_{j=1}^{\infty}\left\{\frac{\phi^j\left(\left|\Delta_{gg'}\right|\right)}{j!}\left(\frac{\theta_g}{np_{g'}}\right)^j\right\}\leq 0$$

where, for $b=\{g'',g'\}$, $\phi^j\left(\left|\Delta_{gb}\right|\right)$ is the *j*’th derivative of ϕ with respect to $\left(\theta_g/np_b\right)$

evaluated at $\left|\Delta_{gb}\right|$. Three distinct cases can be identified if the ordering of the groups is transitive: (I) if $H_g \succ H_{g''} \succeq H_{g'}$ then $\partial\left|\Delta_{gg''}\right| \geq 0$ and $\partial\left|\Delta_{gg'}\right| \leq 0$ such that linearity of ϕ is required to ensure that the net change in stratification with respect to group *g* is non-positive. In particular, linearity implies that $\phi\left(S_g\right)$ is invariant to the membership swap since $\partial\phi\left(S_g\right) = \beta_1\left(p_{g''}\left(\theta_g/np_{g''}\right) - p_{g'}\left(\theta_g/np_{g'}\right)\right) = 0$ if $\phi\left(S_g\right) = \beta_0 + \beta_1 S_g$. Otherwise, $\partial\phi\left(S_g\right)$ may be either positive or negative depending on the group populations shares $p_{g''}$ and $p_{g'}$; (II) if $H_{g''} \succeq H_{g'} \succ H_g$ then $\partial\left|\Delta_{gg''}\right| \leq 0$ and $\partial\left|\Delta_{gg'}\right| \geq 0$, such that the need for ϕ to be linear follows by a similar argument based on the invariance of $\phi\left(S_g\right)$; (III) if $H_{g''} \succeq H_g \succeq H_{g'}$ then both $\partial\left|\Delta_{gg''}\right| \leq 0$ and $\partial\left|\Delta_{gg'}\right| \leq 0$ such that the required non-positivity of $\partial\phi\left(S_g\right)$ follows immediately.

Dropping transitivity of the ordering admits a further case: (IV) if $H_{g''} \succ H_{g'}$ and $H_{g'} \succ H_g$ but $H_g \succ H_{g''}$ then both $\partial\left|\Delta_{gg''}\right| \geq 0$ and $\partial\left|\Delta_{gg'}\right| \geq 0$ by construction, thereby opening up the possibility for violations of the between-group membership principle even if ϕ is linear. For example, if $H_{g''} = (5,4,4,1)$, $H_{g'} = (4,3,3,3)$ and $H_g = (5,5,5,4,2,2,2,2)$ then $\Delta_{g'g''} = 3/8$ and $\Delta_{gg'} = 1/32$ but $\Delta_{gg''} = -1/32$. Hence $S = 1/32$ for any linear ϕ, given $\Delta_{gg} = \Delta_{g'g'} = \Delta_{g''g''} = 0$ by definition, but this rises to 3/64 if the healthiest person in group g'' swaps groups with the healthiest person in group *g′*. In general, a membership swap will lead to violations of the principle in the nontransitive case if the change in *S* due to the increase in pairwise stratification with respect to groups with profiles better than g'' but worse than g′ more than offsets the change due to the reduction in pairwise stratification with respect to both the two groups involved in the swap and all other groups with health profiles better than g′ and worse than g''. For the simplest case of a population that consists only of the 3 groups forming the statistical preference cycle then this condition implies that violations will occur if:

$$p_g\left(p_{g'}\partial\left|\Delta_{g'g}\right|+p_{g''}\partial\left|\Delta_{g''g}\right|\right)>-p_{g'}p_{g''}\partial\left|\Delta_{g'g''}\right| \tag{5}$$

where the identity of group *g* may be arbitrarily chosen to be any one of the three groups and with the membership swap then assumed to takes place between the other two groups. The strict inequality will hold if:

$$2n_g\theta_g>\left(n_{g'}\theta_{g'}+n_{g''}\theta_{g''}\right) \tag{6}$$

where, for all $a\in G,\ n_a\theta_a=\left(\sum_{h=h^-}^{h^+-1}N\left(H_a=h\right)\right)+\left(\sum_{h=h^-+1}^{h^+}N\left(H_a=h\right)\right)>0$ is equal to the sum of the number of group *a* members in the overlapping closed intervals $h=\left[h^-,h^+-1\right]$ and $h=\left[h^-+1,h^+\right]$. Thus the inequality must hold for at least one choice of group *g* from the three groups, implying violation of the between-group exchange principle if the associated membership swap between the other two groups is both feasible and preserves the ordering of groups, *unless* $n_g\theta_g=n_{g'}\theta_{g'}=n_{g''}\theta_{g''}$. More generally, the condition implies that violations will occur if $2\sum_{g\in G^{\oplus}_{g''g'}}n_g\theta_g\geq\left(n_{g'}\theta_{g'}+n_{g''}\theta_{g''}\right)+2\sum_{g\in G^{\otimes}_{g''g'}}n_g\theta_g$, where $G^{\oplus}_{g''g'}$ is the sub-set of groups with profiles better than g'' but worse than g′ and $G^{\otimes}_{g''g'}$ consists of groups with profiles better than g′ and worse than g''.

Finally, no further loss of generality is entailed by restricting ϕ to be the identity function since all linear ϕ generate the same value of the stratification index *S*. Interest may therefore be confined to the index proposed by Allanson (2017a):

$$S=\sum_{g=1}^{G}\sum_{g'=1}^{G}p_gp_{g'}\left|\Delta_{gg'}\right|=\sum_{g=1}^{G}p_g\left(\sum_{g'=1}^{G}p_{g'}\left|\Delta_{gg'}\right|\right)=\sum_{g=1}^{G}p_gS_g; \tag{5}$$

The bounds of S in (5) are the closed interval $[0,(1-\sum_g p_g^2)]$: dividing through by $(1-\sum_g p_g^2)$ yields a normalised index that is the population-weighted mean level of pairwise stratification between all mutually distinct groups, with a minimum value of zero and a maximum of one. This normalised index is both linear homogenous and translatable in the sense that a proportional change in all pairwise disparities $\left|\Delta_{gg'}\right|$ between distinct groups will lead to an equiproportionate change in the index, whereas a unit change in all such pairwise disparities would lead to a unit change in the index.

5. Conclusion

The paper provides a characterisation of the Allanson (2017a) headcount stratification index, which is defined as the population-weighted mean difference in the probabilities that the healthier of any randomly chosen pair of individuals will be from the group with the better rather than worse population health. The approach is motivated by the concept of statistical preference (De Schuymer et al., 2003) which, following Dubois et al. (2003), can be shown to provide the only basis for the pairwise comparison of group health profiles if individual health is only ordinal measurable.

We further propose a between-group exchange principle as the fundamental property of a headcount stratification index, with this playing the same role as the Pigou-Dalton principle of transfers in the measurement of inequality. It is shown that the principle restricts interest to the Allanson index if the ordering of groups is transitive, although violations are all but inevitable for at least some membership exchanges within subsets of groups forming statistical preference cycles of order 3. However transitivity may be expected to hold in most empirical work if the shape of the group health profiles is not too irregular, with Allanson (2017a) obtaining a strict total order of regional health profiles in the United Kingdom.

Our between-group exchange principle is a ‘global’ condition unlike the ‘local’ version identified by Reardon (2009) as a characteristic of a class of vertical segregation indices that measure the extent to which ordinal variation within subgroups is less than the total ordinal variation in the population. Allanson (2017b) labels an analogous property in the measurement of income-related health stratification as a “health status” exchange condition but it is perhaps more natural to think of a switch in group membership between two individuals, for example by migration between regions, than in health status. Moreover, a so-called ‘Hammond’ transfer that merely reduced the spread of health between the two individuals (cf. Gravel et al., 2020) with no exchange in group membership would not in general be guaranteed to reduce stratification since *S* may increase as a result if, for example, the stratification of the first group did not change while that of the less healthy group increased in relation to other groups with even worse health profiles.

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